Math 244
Stokes and Divergence Worksheet

1. Determine whether the following integrals can be computed using Stoke's (or Green's) Thoerem, Divergence Thoerem, or the Fundamental Theorem of Line Integrals and fully justify why each theorem can be applied

   (a) $\iint_\Sigma \langle x^3 + \ln(2+\cos(z)), y^3, z - \sqrt{1+x^2}\rangle \cdot d\vec{S}$ where Σ is the sphere of radius 3 centered at the origin

   (b) The line integral of $\vec{F} = (y - y^3)\vec{j} + e^z\vec{k}$ over the curve C parameterized by $\vec{r}(t) = \langle \sqrt{1+\cos(t)}\cos(t), \sin(t), \ln(1+e^t)\rangle$, $0 \le t \le \pi$

   (c) Calculate $\int_C \vec{F}\cdot\vec{T}ds$ where $\vec{F} = x^2y\vec{i} + \frac{1}{3}x^3\vec{j} + xy\vec{k}$ and C is the curve of intersection of the hyperbolic parabaloid $z = y^2 - x^2$ with the cylinder $x^2 + y^2 = 1$ such that C has counter clockwise orientation about the z-axis.

   (d) For any $f : \mathbb{R}^3 \to \mathbb{R}$ that has continuous second partial derivatives then what theorem can be used to prove that $\int_C (f\nabla f)\cdot d\vec{r} = 0$ for C a closed curve in $\mathbb{R}^3$.

   (e) Computing the outward flux of $\vec{F} = \langle xyz, x^2 - y^3 + z^3, \ln(z^2+1) - xy^4\rangle$ through the surface of the rectangle $[-1,2]\times[3,4]\times[-2,2]$

(f) Which theorem can one use to prove that $\iint_\Sigma \text{curl}(\vec{F}) \cdot d\vec{S} = 0$ for any nice enough vector field $\vec{F}$ and any closed surface Σ?

(g) $\iint_\Sigma (z \arctan(y^2)\vec{i} + z^3 \ln(x^2+1)\vec{j} + z\vec{k}) \cdot d\vec{S}$ where Σ is the part of the parabaloid $z = 2 - x^2 - y^2$ that lies above the plane $z = 1$ and is oriented upwards.

(h) The work done by the vector field $\vec{F} = \langle e^y, xe^y, (z+1)e^z \rangle$ over the curve of intersection between the two parabaloids $z = 4 - x^2 - y^2$ and $z = x^2 + y^2$ contained in the first octant such that the curve has counterclock wise orientation about the z-axis.

2. Now prove or compute all parts of the previous problem

(a) $\iint_\Sigma \langle x^3 + \ln(2 + \cos(z)), y^3, z - \sqrt{1+x^2} \rangle \cdot d\vec{S}$ where Σ is the sphere of radius 3 centered at the origin

(b) The line integral of $\vec{F} = (y - y^3)\vec{j} + e^z\vec{k}$ over the curve C parameterized by $\vec{r}(t) = \langle \sqrt{1 + \cos(t)} \cos(t), \sin(t), \ln(1 + e^t) \rangle$, $0 \leq t \leq \pi$

(c) Calculate $\int_C \vec{F}\cdot\vec{T}ds$ where $\vec{F} = x^2y\vec{i}+\frac{1}{3}x^3\vec{j}+xy\vec{k}$ and C is the curve of intersection of the hyperbolic parabaloid $z = y^2 - x^2$ with the cylinder $x^2 + y^2 = 1$ such that C has counter clockwise orientation about the z-axis.

(d) For any $f : \mathbb{R}^3 \to \mathbb{R}$ that has continuous second partial derivatives then what theorem can be used to prove that $\int_C (f\nabla f)\cdot d\vec{r} = 0$ for C a closed curve in $\mathbb{R}^3$.

(e) Computing the outward flux of $\vec{F} = \langle xyz, x^2 - y^3 + z^3, \ln(z^2 + 1) - xy^4\rangle$ through the surface of the rectangle $[-1, 2] \times [3, 4] \times [-2, 2]$

(f) Which theorem can one use to prove that $\iint_\Sigma \text{curl}(\vec{F})\cdot d\vec{S} = 0$ for any nice enough vector field $\vec{F}$ and any closed surface Σ?

(g) $\iint_{\Sigma} (z \arctan(y^2)\vec{i} + z^3 \ln(x^2+1)\vec{j} + z\vec{k}) \cdot d\vec{S}$ where Σ is the part of the parabaloid $z = 2 - x^2 - y^2$ that lies above the plane $z = 1$ and is oriented upwards.

(h) The work done by the vector field $\vec{F} = \langle e^y, xe^y, (z+1)e^z \rangle$ over the curve of intersection between the two parabaloids $z = 4 - x^2 - y^2$ and $z = x^2 + y^2$ contained in the first octant such that the curve has counterclock wise orientation about the z-axis.